

Simulation of consolidation and liquid flow in a farm tower silo

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Tang, J. and Jofriet, J. C. 1989. **Simulation of consolidation and liquid flow in a farm tower silo.** Can. Agric. Eng. 31: 167-174. The storage of relatively wet silage in a tower silo often causes the silage to become saturated. Silage juice is expelled and wall pressures increase substantially. The magnitude of these increased wall pressures are not well known. This paper presents a procedure to simulate the consolidation of silage in a tower silo and the liquid flow in the saturated zone. The output from the simulation includes the saturation level in the silage, the fiber and liquid pressures and the total amount of silage juice effluent.

The 30-d simulation of a full-scale test by 't Hart et al. (1980) shows that the procedure provides satisfactory results. A detailed comparison of settled height of silage and of fiber and liquid pressures are presented. The results of the simulation show the importance of good silo drainage in reducing the liquid pressures in the saturated silage zone.

INTRODUCTION

When high-moisture silage (>65%) is stored in a tower silo the consolidation pressures in the bottom of the silo are such that the silage becomes saturated. This results in substantially higher wall pressures than is the case with dryer materials.

Three time-dependent processes are involved when high-moisture silage is stored in a tower silo equipped with drains. They are silo filling, silage consolidation, and juice drainage. All three have an effect on the magnitude of hydrostatic pressures the silo wall will be subjected to over time and all three are interdependent.

The silo-filling process is determined by the crop-harvesting management. The consolidation process is governed by the physical properties of the silage and the interaction between the silage material and the silo wall. It is characterized by a decrease in the height of the silage mass in the silo and an increase in silage density. Due to the consolidation, silage gets saturated in parts of the silage mass, and a saturation zone develops. Further consolidation tends to raise the height of the saturation level. In the drainage process, liquid flows through the silage material and drains out of the silo through the drain under the action of gravity and the fluid pressure gradient. As a result of the complex interaction of these three time-dependent processes the prediction of the saturation level and the liquid pressure is extremely difficult.

The Canadian Farm Building Code (CFBC) (Associate Committee on the National Building Code (ACNBC) 1983) suggests in an appendix that the saturation depth may be taken as 30 m for 65% forage, 16 m for 70% and 11 m for 75% material. However, the CFBC warns that "there is no precise guideline for determining the saturation depth". The next version of the CFBC (ACNBC 1990) will provide a formula for predicting the saturation depth of a silo filled with wet forage

$$H_s = 160 - 2M - D \quad (1)$$

in which H_s is the saturation depth in m, M is the moisture content of the forage in percent wet basis (WB), and D is the diameter of the silo in m. However, Eq. 1 is not based on much more solid information than the provisions in the present Canadian Farm Building Code (ACNBC 1983).

A comprehensive computer-assisted procedure was developed to simulate the three processes in order to predict the time-dependent change of the height of the saturation zone and the magnitude of the lateral pressure. An outline of the procedure will be given in this paper. The relevant theories will be discussed. The simulation results will be compared with those of a full-scale experiment carried out in the Netherlands.

METHOD OF SIMULATION

A body of silage can be considered a particulate medium in which the particles have a cell-like structure. Wood (1970), 't Hart et al. (1980), Nilsson (1982) and Tang and Jofriet (1988a,b) have shown that saturation of the particulate medium occurs well before all gases are expelled from the voids. Consequently, consolidation will take place through the dissipation of excess pore pressure developed as a result of load application and of a further reduction of gas content.

Lau and Jofriet (1988) modelled a body of silage as a particulate medium of solid particles using Biot's (1941) general theory of three-dimensional consolidation. They concluded that excess pore pressures are small relative to the hydrostatic pressures and that the effect is relatively short-term compared to the typical rate of loading. Excess pore pressures were neglected in this study based on these earlier findings.

The simulation procedure performs four major operations, and these operations are represented symbolically in Fig. 1 by four blocks. The simulation begins with the adding of discrete layers of silage of a given mass according to a filling record provided as input. The consolidation of the layers in the silo is simulated in block 1 using the lamina approach proposed by Wood (1970). The dry matter density and the fiber pressure are calculated for each lamina.

The saturation criterion developed by Tang and Jofriet (1988a) is used to determine for each lamina if saturation occurs. If one or more layers have become saturated, an effluent production model (Tang and Jofriet 1988b) is used to predict the amount of juice expelled from the saturated layers, and also the water-holding capacity of layers drained previously. Permeability models developed by Tang (1987) are used in block 1 to predict the current vertical and horizontal permeabilities. These permeability values are transferred to block 2.

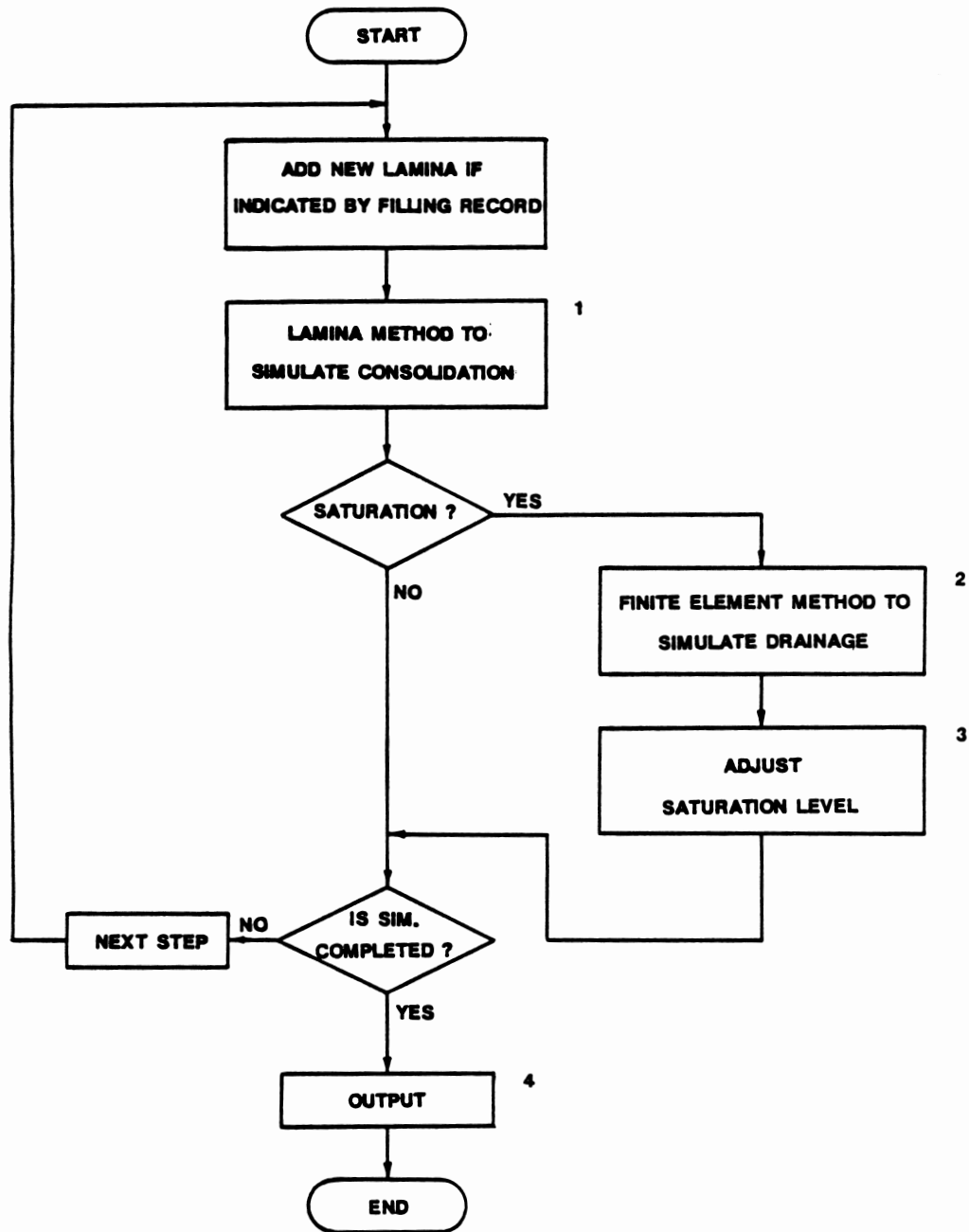


Figure 1. Flow chart of the simulation.

The drainage process is simulated in block 2 by the finite element method of analysis. In this analysis the amount of juice drained out of the silo during the current time step is calculated. The fluid pressures on the silo wall are also computed in block 2. Block 3 receives information from blocks 1 and 2 and adjusts the saturation level for the next time step. If the moisture content of silage is low and no saturation appears, the simulation will bypass blocks 2 and 3.

The above operations are performed at each time step of the simulation thus providing a numerical coupling of the processes. The interaction between the procedures in blocks 1, 2, and 3 is shown in Fig. 2. Block 4 processes all results at the completion of the simulation and prints the output. The fluid pressure calculated by finite element method is superimposed on the fiber pressure determined by lamina method to obtain total lateral

pressure. Details of the lamina method used in block 1 and the finite element method in block 2 follow.

The Lamina Method

The lamina method has been used effectively in simulating consolidation of silage materials in tower silos (Wood 1970; Arnold 1974; 't Hart et al. 1980; Nilsson 1982; Lau 1983). It has also been employed in the present procedure. The outcomes of the lamina simulation are the dry matter density of the silage, the height of the silage mass, and the fiber (effective) pressure exerted by the silage on the silo wall.

The following assumptions were used in the lamina method:

(1) The vertical stress is uniform in horizontal planes in the silage.

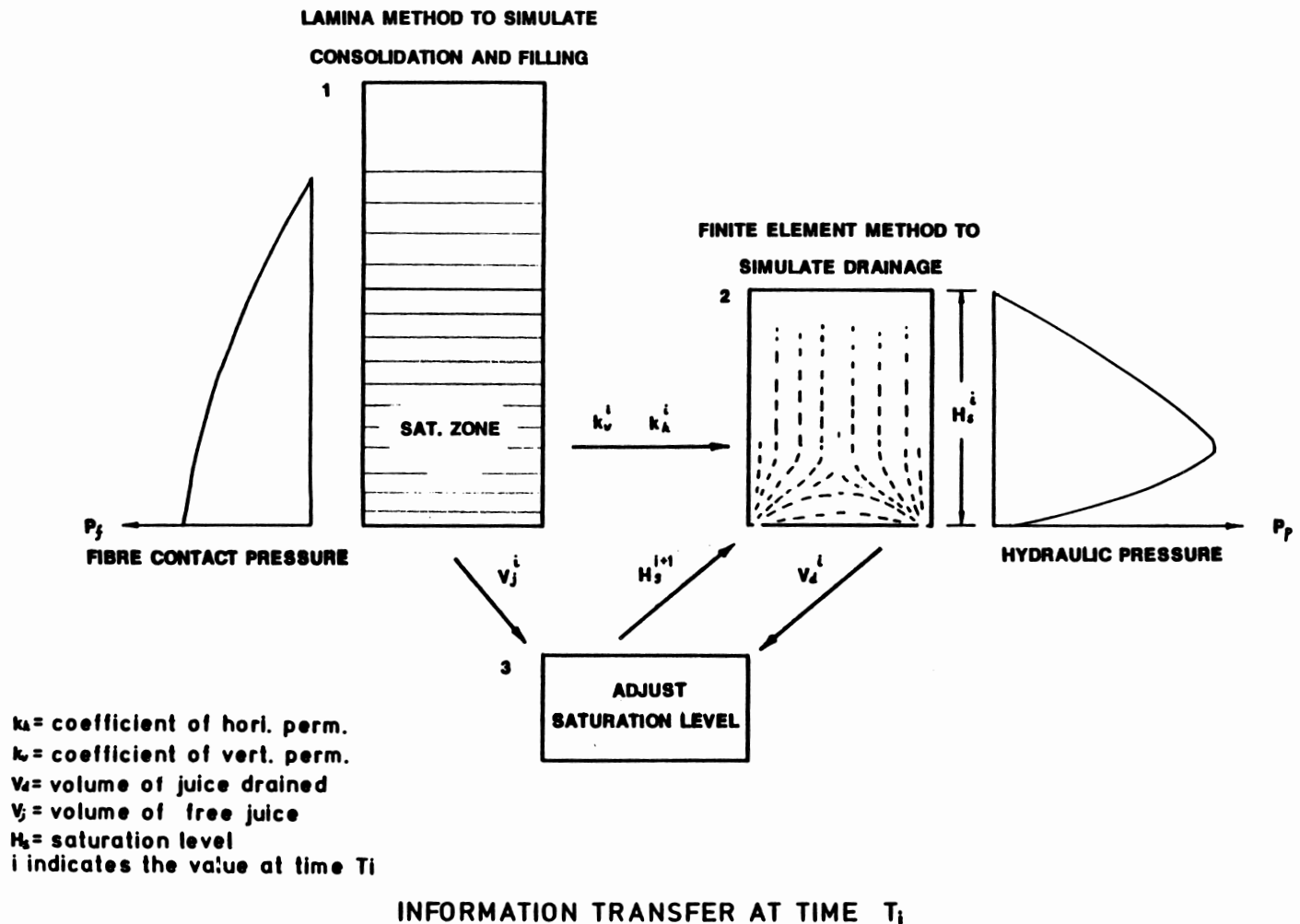


Figure 2. Information transfer among blocks 1, 2 and 3 at simulation time t_i .

- (2) The density is uniform in each silage lamina.
- (3) The coefficient of friction between silage and silo wall and the pressure ratio are constant.
- (4) The silage in the silo is in a state of static equilibrium.
- (5) The appearance of silage juice does not affect the consolidation.

The equilibrium of vertical forces in the vertical (z) direction acting on a horizontal layer of thickness Δz yields.

$$P_v = (\rho_b g - \beta P_v) \Delta z / (1 + 0.5 \beta \Delta z) \quad (2)$$

where:

$$\beta = 4 \mu K / D,$$

μ is the coefficient of friction,

K is the ratio of horizontal to vertical pressures,

P_v is the vertical pressure acting on the top of the lamina,

g is the acceleration due to gravity, and

ρ_b is the bulk density of the silage in the lamina.

The dry matter density of the silage, ρ_d , can be predicted in terms of vertical pressure P_v and the duration of consolidation, t (Wood 1970; Arnold 1974; 't Hart et al. 1980; Nilsson 1982; Jofriet et al. 1982; Negi and Jofriet 1984), that is

$$\rho_d = \rho_d(P_v, t) \quad (3)$$

The relationship between bulk density, ρ_b , and dry matter density, ρ_d , is

$$\rho_b = 100 \rho_d / (100 - M) \quad (4)$$

where M is the moisture content, in percent (WB).

Due to the consolidation, the thickness of a lamina with a given mass of silage decreases with time. If the initial bulk density of the silage in the lamina is ρ_{b0} , and the initial thickness of the lamina is Δz_0 , the thickness of the lamina can be calculated by

$$\Delta z = \Delta z_0 (\rho_{b0} / \rho_b) \quad (5)$$

In the simulation program Eqs. 2 – 5 are solved iteratively to obtain vertical pressure P_v , dry matter density ρ_d , and the thickness Δz for each lamina of silage at every time step starting at the top layer of the silage in the silo. The lateral fiber pressure on the silo wall is calculated from $K P_v$.

As all density models (such as Eq. 3) are based on constant load consolidation tests, they can not be used directly for the situation in which variable load is involved as is the case in a silo. It is known that silage is a strain-hardening material (Jofriet et al. 1982). Therefore, the strain-hardening approach (Hult 1966, p. 33) is employed in the simulation model so that the density model in Eq. 3 can be used for the laminae under increasing vertical load.

The Finite Element Method

The finite element method was used to simulate the drainage

of silage juice from the silage mass. The following assumptions were made:

- (1) Silage juice is a homogeneous incompressible Newtonian fluid.
- (2) Darcy's law is valid.
- (3) The coefficient of vertical and horizontal permeabilities are constant in horizontal planes.
- (4) The flow is axisymmetrical.
- (5) During each simulation time step the flow is steady, and the coefficients of vertical and horizontal permeabilities are constant.
- (6) The silo wall and the bottom are impermeable; the pressure at the top of the saturation zone and at the drain is atmospheric.

Based on the first five assumptions, the governing equation for the juice flow in a cylindrical coordinate system can be written as (Reddy 1984):

$$k_h \left[\frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} \right] h + \frac{\partial}{\partial z} \left[k_v \frac{\partial}{\partial z} \right] h = 0 \quad (6)$$

where:

k_v and k_h are the horizontal and vertical permeabilities, h is the total hydraulic head, $p/\rho g + z$, p is the fluid pressure, g is the unit weight of the fluid. The horizontal and vertical permeabilities in a saturation zone are not constant but vary inversely with the degree of consolidation (Tang 1987). Thus the permeabilities decrease with the depth of silage from the top of the saturation zone.

According to assumption 6, the boundary conditions are: $p = 0$ at the top of the saturation zone and at the drain;

$\partial h/\partial r = 0$ at the interface between the silage and the silo wall; and $\partial h/\partial z = 0$ at the bottom of the silo.

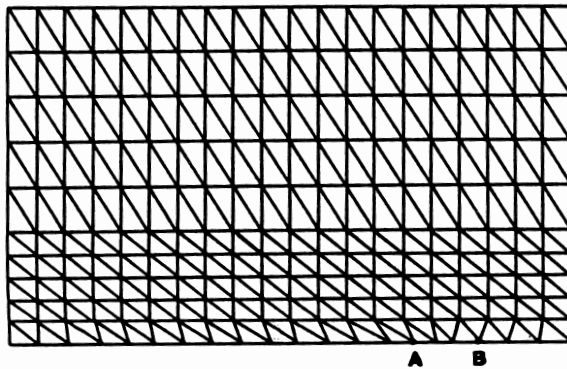
The finite element method of analysis was used to solve Eq. 6 for the hydraulic head at each time step of the simulation. The finite element method package, FEMPAC (Gustafson 1977) was adapted and included in the present simulation package.

To cope with the problem of changes in the height of the flow domain, a subroutine was included in the simulation package to generate a linear triangular element grid at each time step to suit the height of the present saturation zone. In the radial direction 20 elements of about equal length were used (see Fig. 3). In the vertical direction, half of the elements were concentrated in the bottom 1/3 portion of the height of the grid to guarantee sufficient accuracy of the solution in the bottom portion of the silo where there are relatively high fluid pressure gradients. The number of the elements depends on the height-to-radius ratio of the saturation zone. There are 10, 20, and 30 element divisions respectively for a height-to-radius ratio of less than 0.4, between 0.4 and 2, and greater than 2. As a result, the aspect ratio of all the elements generated by the program are between 0.3 and 5 for saturation zone heights from about 0.3 – 20 m. This will ensure a reasonable accuracy. Two of the computer-generated grids are shown in Fig. 3. A-B indicates the position and width of the drain.

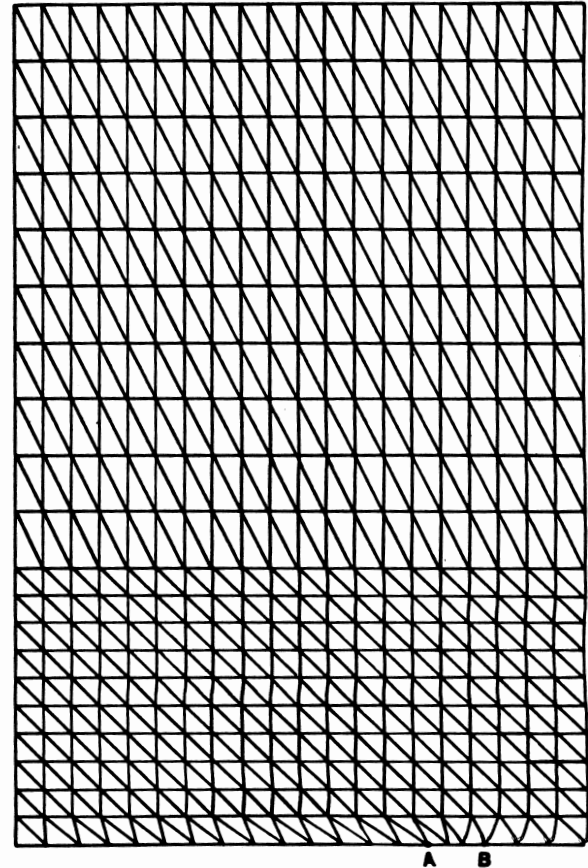
SIMULATION

The simulation program was written in Fortran 77. The program requires the following input:

- (1) The diameter and the height of the silo, and the position and width of the drain.
- (2) The moisture content M , and the pressure ratio K of the



MESH 1



MESH 2

Figure 3. Layout of elements for flow simulation.

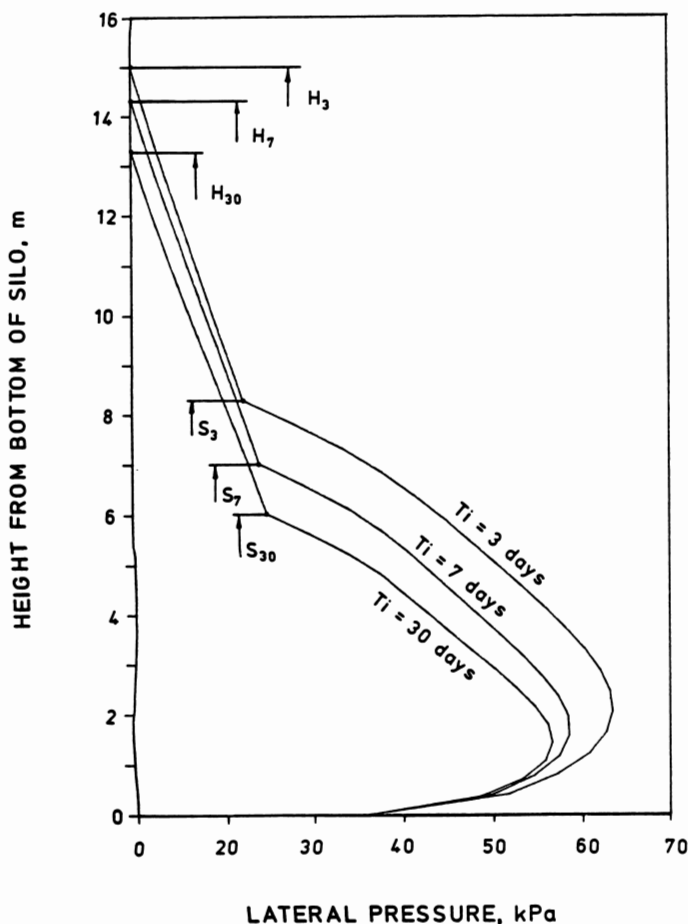


Figure 4. Simulated lateral pressures vs height from the bottom of a 6.19-m-diameter steel silo with 73.9% moisture content whole plant corn silage.

silage, the friction coefficient μ between the silage and the silo wall, and the density-pressure-time model for the silage.

(3) The coefficients of vertical and horizontal permeabilities of the silage.

(4) The control parameters for the simulation such as the time step, the overall simulation period, in days, and the maximum allowable initial thickness, in metres, of the laminae.

(5) Daily silo filling records in terms of silage mass, kg.

The simulation program reads in the daily filling records, and calculates the volume of each load of silage using the initial dry matter density ρ_0 from the density-pressure-time model. The silage material loaded each day is equally divided into a number of layers with an initial thickness not exceeding the maximum allowable initial thickness.

The simulation carries out a complete analysis of consolidation and liquid flow for each of the specified time steps, as illustrated in Fig. 1. The information that is available from the simulation at each time step is the saturation level, the settled height of the silage, and the fiber and liquid pressures on the wall. The amount of output can be reduced by setting output selection parameters. The total volume of effluent from the consolidated silage is also calculated.

The setting of the output selection parameters must be such to obtain all important information without being overwhelmed with output. The program aids in this by a trigger that will

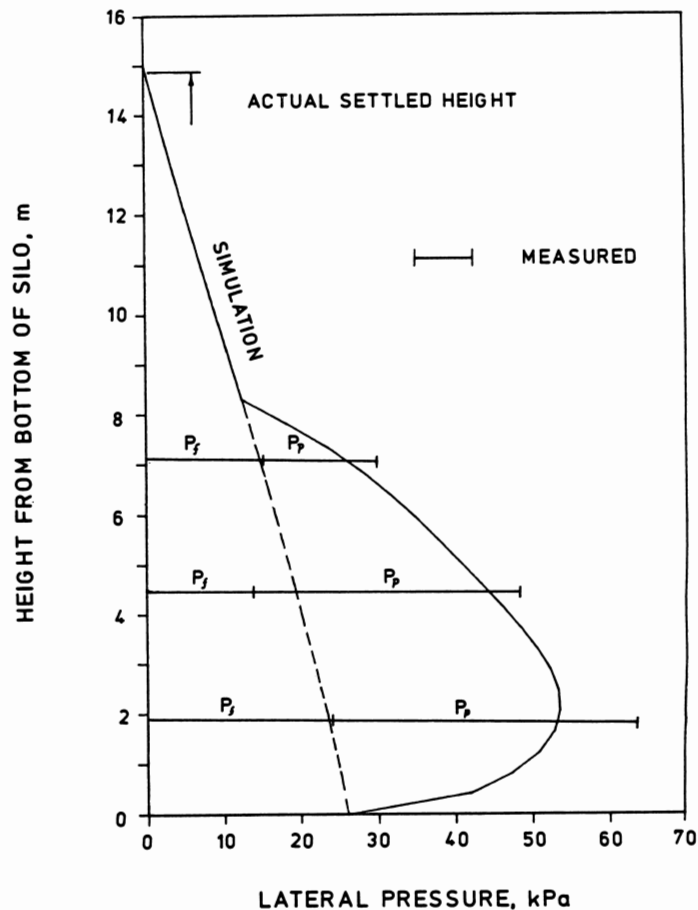


Figure 5. Comparison between measured and simulated pressures 3 d after filling.

automatically cause maximum wall pressures to be output.

VERIFICATION OF THE SIMULATION PROCEDURE

The verification of the simulation procedure was carried out with whole-plant corn silage material, 73.9% moisture content, stored in a 6.19 × 18.15-m steel tower silo. The simulation duplicated as closely as possible all parameters of a full scale experiment performed by 't Hart et al. in 1979 ('t Hart et al. 1980). The simulation results were compared with all available observations.

't Hart et al. (1980) took measurements of the total lateral pressure, the liquid pressure, the load on the floor, and the friction on the silo wall. They also measured the settled height of the silage mass, and the effluent drained from the silo. Four pressure-measuring panels were installed in the silo wall, 90° apart, at three elevations (7.125, 4.535 and 1.94 m from the silo floor), for a total of 12 sensors. The total lateral pressure and fluid pressure were reported at 3, 7, 14, 21, and 30 d after filling of the silo. Readings from four sensors at the same level were averaged.

The silo was provided with a test floor supported on four load cells to register the vertical load on the floor. The silage effluent was drained through the gap between the silo wall and the test floor. The width of this gap was not reported. A value of 0.02 m was used in the simulation. A parametric study showed that the

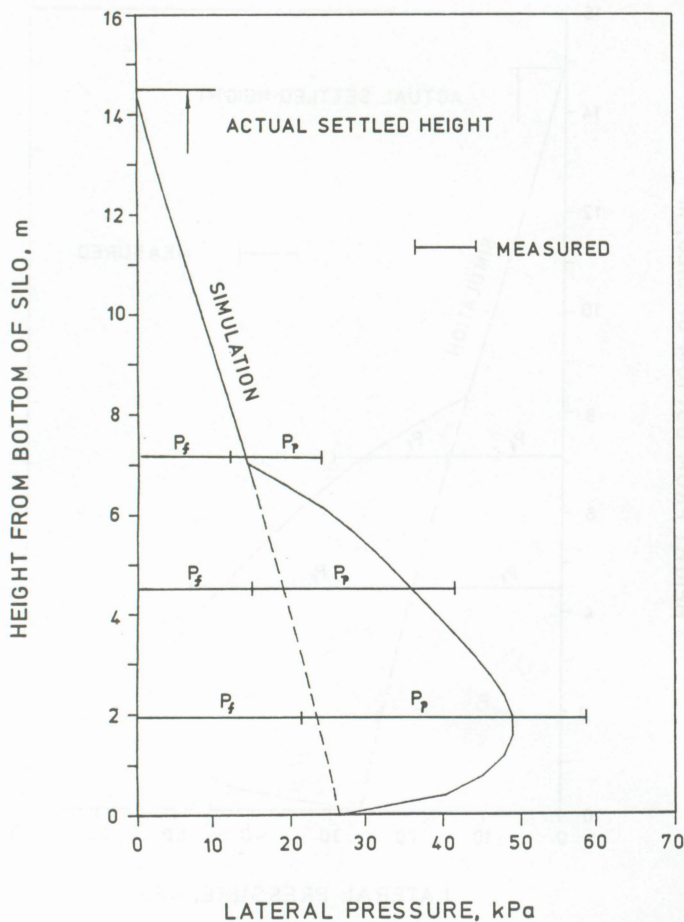


Figure 6. Comparison between measured and simulated pressures 7 d after filling.

simulation is not very sensitive to width of the drain.

A total of 404 tons of 73.9% (WB) moisture content whole-plant corn silage was loaded in the silo on the 1st, 2nd, and 5th days after the start of the test. The weight of each load was not reported. The authors assumed that an equal amount of silage was loaded on each of those three filling days.

In order to simulate the test condition of 't Hart et al. (1980) as closely as possible, their density model

$$\rho_b = (a_1 + a_2 \log(t))(a_3 + a_4 \log(t))(\log(P_v))^2$$

was incorporated in the simulation. Values for a_1 , a_2 , a_3 and a_4 were 130.5, 2.11, 33.77, and 7.02, respectively ('t Hart et al. 1980). The coefficient of friction, μ , and the pressure ratio, K , were chosen to be 0.4 and 0.33 from the same source.

The simulation control parameters were: simulation period = 30 d; simulation time step $\Delta t = 0.1$ d; the maximum initial thickness of the layers in the lamina method was 1 m.

RESULTS AND DISCUSSION

The simulated total lateral pressures exerted by the silage on the wall 3, 7 and 30 d after the filling of the silo have been plotted in Fig. 4. H_3 , H_7 and H_{30} indicate the settled heights of the silage, and S_3 , S_7 and S_{30} represent the saturation levels. The simulation shows that the settled height decreases with storage time and that the saturation level is highest at 3 d.

The pressure profiles in Fig. 4 indicate that the lateral

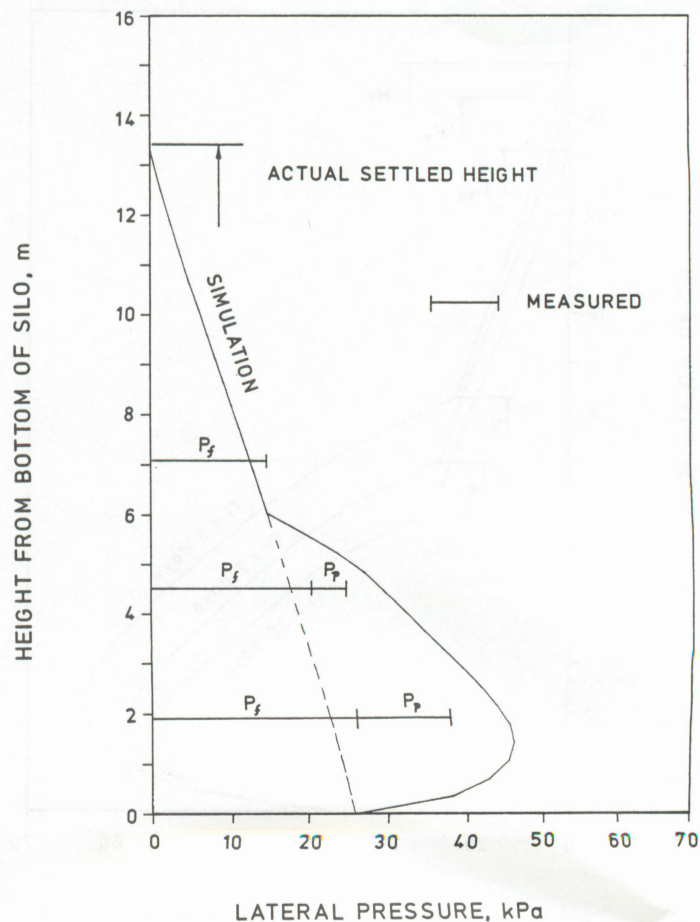


Figure 7. Comparison between measured and simulated pressures 30 d after filling.

pressure increases dramatically below the saturation level. The drain relieves liquid pressure close to the bottom of the silo, and hence reduces the total lateral pressure at that location. The reduction in total pressure and saturation level due to drainage is greater than their increases caused by consolidation.

The simulation results are compared with the averaged measurements of 't Hart et al. (1980) at 3, 7, 30 d after the filling in Figs. 5, 6 and 7, respectively. Settled silage height, fiber pressure, liquid pressure and effluent quantity will be considered. The simulated values of fiber pressure below the saturation level are indicated by dash lines, the total lateral pressures by solid ones. The values of liquid pressure are the difference. The observed fiber pressures are indicated by P_f and the liquid pressures by P_l . The total observed lateral pressures are the sum of the fiber and liquid pressures.

The simulated values for the settled height of the silage mass agree extremely well with the measurements. The maximum difference is less than 3%.

The simulated fiber pressures agree reasonably well with the observed ones. The average differences between the simulated values and the measured ones are about 11, 14, and 13% at 3, 7, and 30 d, respectively. The largest values of both simulated and measured total lateral pressures occurred at about 2 m above the silo floor. The differences between the three simulated maximum total lateral pressures and the measured ones are less than 17%.

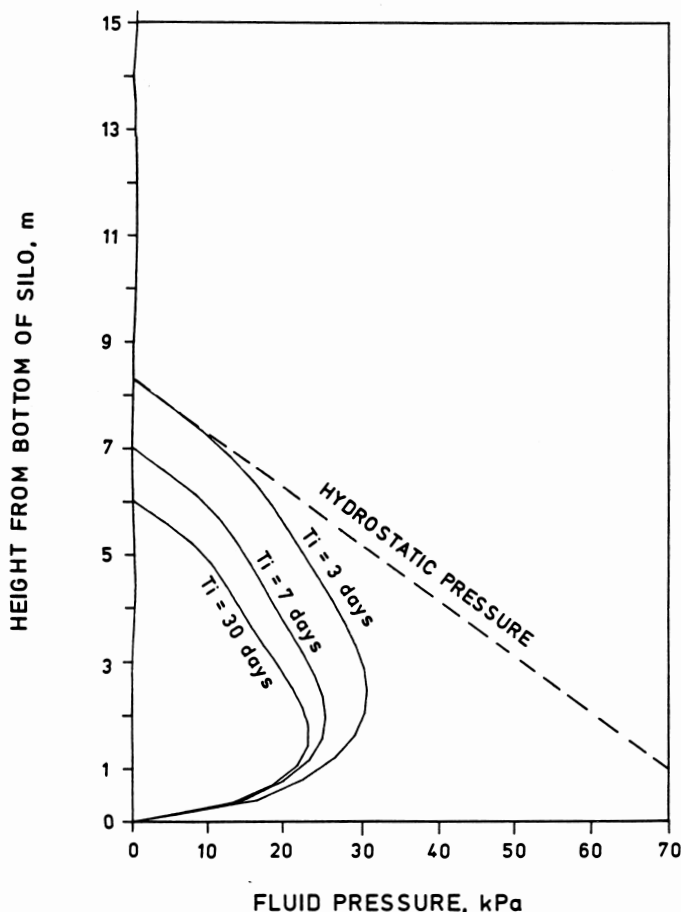


Figure 8. Simulated fluid pressure in a 6.19-m-diameter steel silo with 73.9% moisture content whole plant corn silage.

't Hart et al. (1980) did not report a direct measurement of the saturation level. It is impossible, therefore, to check the simulation results of the saturation level.

The total volume of the effluent production from the silo was computed with the effluent model (Tang and Jofriet 1988b) from the silage density of each lamina at the completion of the simulation. The calculated volume was 37.5 m^3 . This compares very well with the measured value of 36.3 m^3 (private communication with C. 't Hart 1983).

The simulated liquid pressures have been plotted in Fig. 8. The parabolic shape of the liquid pressure diagrams in the saturation zone is the result of the zero fluid pressure boundary condition at the circumferential drain in the bottom of the silo at the silo wall, and the sharp decrease in the permeabilities near the bottom of the silo where the dry matter density of the silage is very large.

None of the observed pressures by 't Hart et al. (1980) showed the reduction in fluid pressure near the floor of the silo. The reason is probably that no sensors were present in the region where the effect of the drain on the fluid pressure is felt. Nilsson (1982) carried out a full-scale experiment in which pressure sensors were installed at an elevation of 0.8 m from the silo floor. The silo content was 62% moisture content grass silage. Nilsson's observed liquid pressure are shown in Fig. 9. Even though a direct comparison between these observations and the simulated results is impossible because of the difference in material, it is interesting to examine qualitatively the liquid pressures obtained from Nilsson's experiment. The parabolic shapes of the simulated liquid pressure diagrams (Fig. 8) are

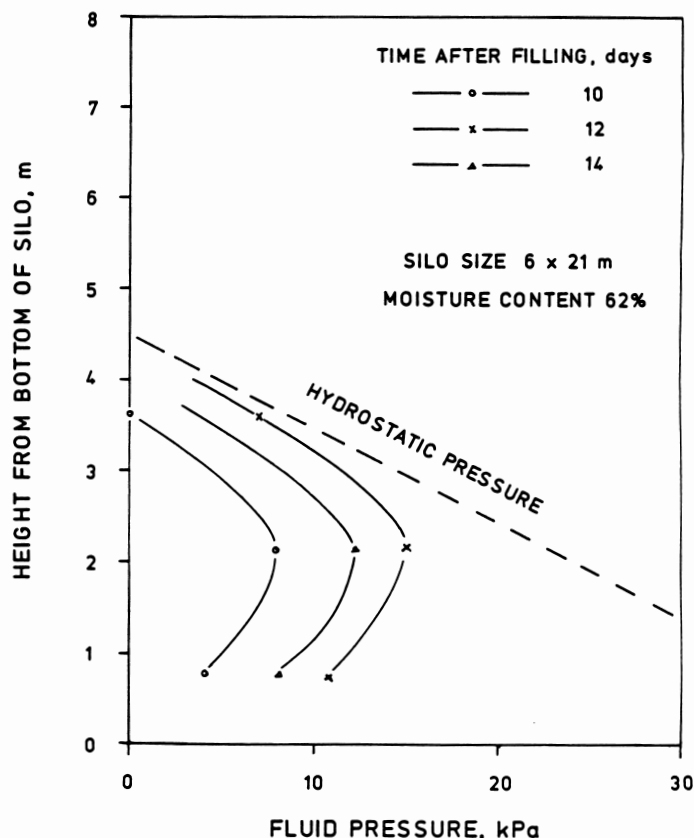


Figure 9. Results of fluid pressure measurements for grass silage in a tower silo with a circumferential drain (Nilsson 1982).

very similar to those observed by Nilsson (1982) in his full-scale measurements (Fig.9).

A comparison of Figs. 8 and 9 also reveals that it took much less time for the height of the saturation zone of whole-plant corn silage to reach its peak (in the first few days after filling) than it did for grass silage (about 12 d). This is probably due to the lower permeabilities of grass silage.

SUMMARY AND CONCLUSIONS

A procedure has been developed to simulate the consolidation of silage in a tower silo and the liquid flow in the saturated zone. The output from the simulation includes the saturation level in the silage, the fiber and liquid pressures and the total amount of silage juice effluent.

A 300-step simulation was carried out of a full-scale test carried out by 't Hart in 1979 with corn silage in a 6.19-m-diameter steel silo. The simulated values for the settled height of the silage mass, the total effluent production from the silo, and the fiber and total lateral pressures agreed well with the full-scale observations. The simulation provided satisfactory results, considering the complex properties of silage materials and the limited filling information.

From the simulation results, as well as from measurements of 't Hart et al. (1980) and Nilsson (1982), two conclusions can be drawn about the effect of the circumferential drain on reducing the hydraulic pressure:

(1) The drainage releases free juice from the saturated silage mass and thus helps to lower the height of the saturated zone.

As a result, the magnitude of the maximum value of liquid pressure on the wall is reduced.

(2) The circumferential drain provides a zero liquid pressure boundary condition at the bottom of the silo wall, and thus forces the liquid pressure to deviate sharply from the hydrostatic pressure line (Figs. 8 and 9), dramatically reducing the maximum value of the hydraulic pressure, and therefore the maximum total lateral pressure exerted on the silo wall by the wet silage material.

ACKNOWLEDGMENT

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