Measurements & Uncertainty

Chp. 1: Intro/Motivation, Chp. 2: Literature Review, Chp. 3: Theory, Chp 4: Experiment
4.1 Setup
4.2 Procedure

4.3 Data Analysis & Uncertainty Propagation

Goals:

1) Show us what happens to a raw data point prior to being reported.

2) Show us where uncertainty to the reported value comes from.

3) Quantify how confident you are in the reported measurements.

How do we quantify confidence? ⇐ This is where you have values as an engineer.

→ It all comes down to statistics & uncertainty, beginning with

Standard Deviation: $\sigma_u = \sqrt{\dfrac{\sum_{i=1}^{N}(u_i - \bar{u})^2}{N-1}}$

N = # of measurements
u_i = measured value
$\bar{u}$ = mean of all measurements
What you report

→ 68% lie within $\pm\sigma$ of $\bar{u}$ ⇐ 1st standard deviation, coverage factor of 1
→ 95% lie within $\pm 2\sigma$ of $\bar{u}$ ⇐ 2nd standard deviation, coverage factor of 2
→ 99.7% lie within $\pm 3\sigma$ of $\bar{u}$ ⇐ 3rd standard deviation, coverage factor of 3

→ You should do this for every measurement @ steady state. You should also do a repeatability test @ several points & factor that in.

→ Example???

* Sure you might be confident that 99.7% of all measurements fall within σ interval, but this is still only a measurement repeatability test.

→ You also need to propogate uncertainties in the values.

→ Calculating uncertainties via Root-Sum Square (RSS) method:

We want to calculate a value to report from raw data: Q ← H, T or η

We want to know the total uncertainty in Q: δQ

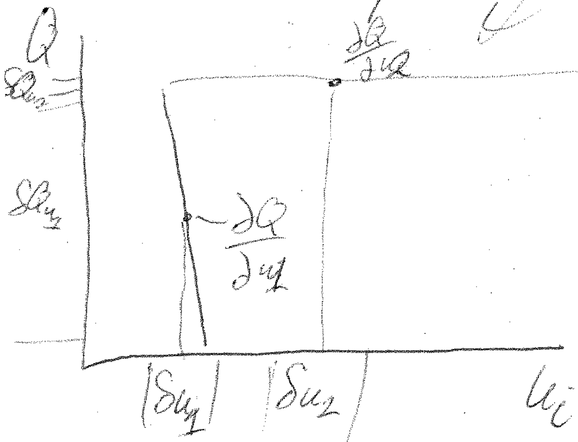
We are measuring independent variables: $u_1, u_2, \ldots, u_N$ ← Voltage, Flowrate, Pressure, Temp...

Each variable contributes uncertainty: δu_i ← published error, user error, limit of resolution..., repeatability

Each variable uncertainty contributes part of the total: δQ_{u_i}

$$\delta Q_{u_i} = \underbrace{\left(\frac{\partial Q}{\partial u_i}\right)}_{\substack{\text{Slope @ a point}\\ \text{of reported value}\\ \text{vs. raw data point}}} \delta u_i$$

Which measured variable uncertainty has a bigger effect on the reported variable?



All of these can be combined through RSS:

$$\delta Q = \sqrt{\delta Q_{u_1}^2 + \delta Q_{u_2}^2 + \delta Q_{u_3}^2 + \ldots}$$

So

$$\delta Q = \sqrt{\left(\left(\frac{\partial Q}{\partial u_1}\right)\delta u_1\right)^2 + \left(\left(\frac{\partial Q}{\partial u_2}\right)\delta u_2\right)^2 + \ldots}$$

↑ Gives total uncertainty in reported value.

Example: Calculate power & uncertainty in a circuit where $V = 100 \pm 2$ volts for 1 measurement

$I = 10 \pm 0.2$ Amps

$P = VI = 1000W \quad \frac{\partial P}{\partial V} = I \ \& \ \frac{\partial P}{\partial I} = V$

the RSS uncertainty in power is then:

$$\delta P = \sqrt{\left(\frac{\partial P}{\partial V} u_V\right)^2 + \left(\frac{\partial P}{\partial I} u_I\right)^2} \Rightarrow \sqrt{(10 \times 2)^2 + (100 \times 0.2)^2}$$

$$\Rightarrow \delta P = 28.3W \text{ so Power } \boxed{P = 1000W \pm 28.3W}$$

Now to do this for multiple measurements & include repeatability We need to determine the RSS of the standard deviation:

$$\sigma_{u,RSS} = \sqrt{\left(\frac{\partial Q}{\partial u_1} \sigma_{u_1}\right)^2 + \left(\frac{\partial Q}{\partial u_2} \sigma_{u_2}\right)^2 + \ldots \text{etc.}}$$

σ from Repeatability $\quad \sigma$ from transmission $\quad$ A/D conversion

Table 6.6 Wheeler & Ganji $\quad$ for large data sets

Random Uncertainty (Precision Error): $\delta P_{u,RSS} = \pm t \frac{\sigma_{u,RSS}}{\sqrt{N}} \Rightarrow \delta P_{u,RSS} = t\,\sigma_{u,RSS}$

Systematic Uncertainty (Bias Error): $\delta B_{u,RSS} = \sqrt{\left(\frac{\partial Q}{\partial u_1} \delta B_1\right)^2 + \left(\frac{\partial Q}{\partial u_2} \delta B_2\right)^2 + \text{etc.}}$

Sensitivity $\quad$ Hysteresis $\quad$ linearity

Total Uncertainty in $Q \Rightarrow \boxed{\delta Q = \sqrt{\delta B_{u,RSS}^2 + \delta P_{u,RSS}^2}}$

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Example: Calculate to a 95% confidence level the uncertainty & power output of an electric motor.

	~~Average~~ Measured Value $\bar{u}$	Bias Error (Systematic Uncertainty)	$\pm 2\sigma$ (Standard deviation)
RPM	1760	3.0	2.5
T (N-m)	20.5	0.3	0.4

$$Power = T \times RPM \times \left(\frac{2\pi}{60}\right) \Rightarrow 20.5\left(\frac{2\pi \times 1760}{60}\right) = 3{,}778\,W$$

$$\frac{\partial P}{\partial T} = RPM \qquad \frac{\partial P}{\partial RPM} = T$$

$$\sigma_P = \sqrt{\left(\frac{\partial P}{\partial T}\sigma_T\right)^2 + \left(\frac{\partial P}{\partial RPM}\sigma_{RPM}\right)^2} \Rightarrow \sqrt{\left(\left(\frac{2\pi\,1760}{60}\right)0.4\right)^2 + \left(\left(\frac{2\pi\,20.5}{60}\right)2.5\right)^2} \Rightarrow \sigma_P = 73.8$$

For 95% confidence (2σ) $t = 2$ from table 6.6

Random error (Precision error): $\delta P_P = 2\sigma_P \Rightarrow \delta P_P = 147.6\,W$

Bias error (Systematic uncertainty): $\delta B_P = \sqrt{\left(\frac{\partial P}{\partial T}B_T\right)^2 + \left(\frac{\partial P}{\partial RPM}B_{RPM}\right)^2}$

$$\Rightarrow \sqrt{\left(\frac{2\pi\,1760}{60}\,0.3\right)^2 + \left(\frac{2\pi\,20.5}{60}\,3.0\right)^2} \Rightarrow \delta B_P = 55.7\,W$$

Total Uncertainty: $\delta Power = \sqrt{\underbrace{55.7^2}_{3102.5} + \underbrace{147.6^2}_{21785.76}} \Rightarrow \delta Power = \pm 157.8\,W$

So $\boxed{Power = 3778\,W \pm 157.8\,W}$ with coverage factor of 2 (95% confidence)

→ From this we could work backwards to determine # of samples needed.