

TOPIC 4: STATISTICAL PROCESS CONTROL

Solutions to Practice Problems

1. $UCL_R = D_4 \cdot \bar{R} = 1.586(4.000) = 6.344$ inches

$LCL_R = D_3 \cdot \bar{R} = 0.414(4.000) = 1.656$ inches

2. a. $\bar{\bar{x}} = 79.75; \bar{R} = 26.5$

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \cdot \bar{R} = 79.750 + 1.023(26.500) = 79.750 + 27.110 = 106.86$

$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \cdot \bar{R} = 79.750 - 1.023(26.500) = 79.750 - 27.110 = 52.64$

b. Yes, because all $\bar{x}$ values fall within the control limits.

3. a.

Sample	Defects	%
1	11	0.091667
2	2	0.016667
3	10	0.083333
4	0	0.000000
5	30	0.250000
6	1	0.008333
7	12	0.100000
8	11	0.091667
9	6	0.050000
10	7	0.058333
11	11	0.091667
12	2	0.016667
13	22	0.183333
14	7	0.058333
15	15	0.125000
16	13	0.108333
17	4	0.033333
18	3	0.025000
19	9	0.075000
20	8	0.066667

$$\bar{p} = \frac{\sum p_i}{k} = \frac{1.533333}{20} = 0.076667$$

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{0.076667(1-0.076667)}{120}} = 0.024288$$

$$UCL_p = \bar{p} + 3\sigma_p = 0.076667 + 3(0.024288) = 0.149531$$

$$LCL_p = \bar{p} - 3\sigma_p = 0.076667 - 3(0.024288) = 0.003803$$

b. Since sample 4, 5, and 13 are out of the control limits, the process is out of control.

$$4. \quad UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 3 + 3\sqrt{3} = 3 + 3(1.732) = 3 + 5.196 = 8.196$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 3 - 3\sqrt{3} = 3 - 3(1.732) = 3 - 5.196 = -2.196 \text{ or } 0$$

5. Total defects = 59

Average defects per order = $\bar{c} = 59/10$

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 5.9 + 3(2.43) = 13.2 \text{ defects/order}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 5.9 - 3(2.43) = -1.39 \text{ or use } 0 \text{ instead since it is impossible to have negative defects per order.}$$