

TOPIC 6: FORECASTING

Solutions to Practice Problems

1. a. $WMA_{August} = \frac{8(50) + 5(100) + 3(75)}{16} = 70.31$
- b. Naïve forecast for August = Actual demand in July = 50
2. a. The exponential smoothing solution is as follows:

Period	Demand	Forecast
1	7	5
2	9	$5 + 0.2(7 - 5) = 5.4$
3	5	$5.4 + 0.2(9 - 5.4) = 6.12$
4	9	$6.12 + 0.2(5 - 6.12) = 5.896$
5	13	$5.896 + 0.2(9 - 5.896) = 6.5168$
6		$6.5168 + 0.2(13 - 6.5168) = 7.81344$

- b. The MAD and MSE can be calculated as the following (the forecasting data are rounded to two decimal places to simplify the calculation):

Period	Demand	Forecast	Error	Error	Error ²
1	7	5.00	2.00	2.00	4.00
2	9	5.40	3.60	3.60	12.96
3	5	6.12	-1.12	1.12	1.25
4	9	5.90	3.10	3.10	9.61
5	13	6.52	6.48	6.48	41.99
6	12	7.81	4.19	4.19	17.56

$$MAD = \frac{\sum_{i=1}^6 |Error|}{6} = \frac{2.00 + 3.60 + 1.12 + 3.10 + 6.48 + 4.19}{6} = \frac{20.49}{6} = 3.42$$

$$MSE = \frac{\sum_{i=1}^6 Error^2}{6} = \frac{4.00 + 12.96 + 1.25 + 9.61 + 41.99 + 17.56}{6} = \frac{87.37}{6} = 14.56$$

3. The solutions to part (a) and (b) are listed in the following table:

Month	Sales	MA (3-month)	MA (5-month)
January	9		
February	7		
March	10		

April	8	$(9 + 7 + 10) / 3 = 8.7$	
May	7	$(7 + 10 + 8) / 3 = 8.3$	
June	12	$(10 + 8 + 7) / 3 = 8.3$	$(9 + 7 + 10 + 8 + 7) / 5 = 8.2$
July	10	$(8 + 7 + 12) / 3 = 9.0$	$(7 + 10 + 8 + 7 + 12) / 5 = 8.8$
August	11	$(7 + 12 + 10) / 3 = 9.7$	$(10 + 8 + 7 + 12 + 10) / 5 = 9.4$
September	12	$(12 + 10 + 11) / 3 = 11.0$	$(8 + 7 + 12 + 10 + 11) / 5 = 9.6$
October	10	$(10 + 11 + 12) / 3 = 11.0$	$(7 + 12 + 10 + 11 + 12) / 5 = 10.4$
November	14	$(11 + 12 + 10) / 3 = 11.0$	$(12 + 10 + 11 + 12 + 10) / 5 = 11.0$
December	16	$(12 + 10 + 14) / 3 = 12.0$	$(10 + 11 + 12 + 10 + 14) / 5 = 11.4$

c. The MADs for the 3-month and 5-month moving average are:

Month	Sales	MA (3-month)	Error (3-month)	MA (5-month)	Error (5-month)
January	9				
February	7				
March	10				
April	8	8.7	0.7		
May	7	8.3	1.3		
June	12	8.3	3.7	8.2	3.8
July	10	9.0	1.0	8.8	1.2
August	11	9.7	1.3	9.4	1.6
September	12	11.0	1.0	9.6	2.4
October	10	11.0	1.0	10.4	0.4
November	14	11.0	3.0	11.0	3
December	16	12.0	4.0	11.4	4.6

$$MAD_{3\text{-month}} = \frac{\sum_{i=1}^9 |Error|}{9} = \frac{0.7 + 1.3 + 3.7 + 1.0 + 1.3 + 1.0 + 1.0 + 3.0 + 4.0}{9} = \frac{17}{9} = 1.89$$

$$MAD_{5\text{-month}} = \frac{\sum_{i=1}^7 |Error|}{7} = \frac{3.8 + 1.2 + 1.6 + 2.4 + 0.4 + 3.0 + 4.6}{7} = \frac{17}{7} = 2.43$$

Therefore, the 3-month moving average is a better forecasting method to use.

4. a. $F_{t+1} = F_t + \alpha(D_t - F_t)$
 $100 + .2(99.5 - 100) = 99.9$
 $99.9 + .2(130 - 99.9) = 105.92$
 $105.92 + .2(190 - 105.92) = 122.736$
 $122.736 + .2(200 - 122.736) = 138.189$
 $138.189 + .2(195 - 138.189) = 149.551$
 $149.551 + .2(125.5 - 149.551) = 144.741$
 $144.741 + .2(160.5 - 144.741) = 147.893$
 $147.893 + .2(189.5 - 147.893) = 156.214$
 $156.214 + .2(172.5 - 156.214) = 159.471$
 $159.471 + .2(236.5 - 159.471) = 174.877$
 $174.877 + .2(118.6 - 174.877) = 163.622$
 $163.622 + .2(101.5 - 163.622) = \mathbf{151.198}$

b. $MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$
 $\left(|130 - 99.9| + |190 - 105.92| + |200 - 122.736| + |195 - 138.189| + \right.$
 $\left. |125.5 - 149.551| + |160.5 - 144.741| + |189.5 - 147.893| + |172.5 - 156.214| + \right.$
 $\left. |236.5 - 159.471| + |118.6 - 174.877| + |101.5 - 163.622| \right) / 11 = 49.22$

c. $MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$
 $\left[(130 - 99.9)^2 + (190 - 105.92)^2 + (200 - 122.736)^2 + (195 - 138.189)^2 + \right.$
 $(125.5 - 149.551)^2 + (160.5 - 144.741)^2 + (189.5 - 147.893)^2 + (172.5 - 156.214)^2 + \left.$
 $(236.5 - 159.471)^2 + (118.6 - 174.877)^2 + (101.5 - 163.622)^2 \right] / 11 = 2995.96$

5. a. $(20 + 21 + 17) / 3 = 19.33$
 $(21 + 17 + 20) / 3 = 19.33$
 $(17 + 20 + 15) / 3 = 17.33$
 $(20 + 15 + 14) / 3 = 16.33$
 $(15 + 14 + 16) / 3 = 15.00$
 $(14 + 16 + 18) / 3 = 16.00$
 $(16 + 18 + 23) / 3 = 19.00$
 $(18 + 23 + 20) / 3 = 20.33$

$$(23 + 20 + 21)/3 = 21.33$$

b. $(.25 * 20 + .25 * 21 + .5 * 17) = 18.75$
 $(.25 * 21 + .25 * 17 + .5 * 20) = 19.50$
 $(.25 * 17 + .25 * 20 + .5 * 15) = 16.75$
 $(.25 * 20 + .25 * 15 + .5 * 14) = 15.75$
 $(.25 * 15 + .25 * 14 + .5 * 16) = 15.25$
 $(.25 * 14 + .25 * 16 + .5 * 18) = 16.50$
 $(.25 * 16 + .25 * 18 + .5 * 23) = 20.00$
 $(.25 * 18 + .25 * 23 + .5 * 20) = 20.25$
 $(.25 * 23 + .25 * 20 + .5 * 21) = 21.25$

c. MAD for a

$$MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$$

$$\left(|20 - 19.33| + |15 - 19.33| + |14 - 17.33| + |16 - 16.33| + |18 - 15| + |23 - 16| + |20 - 19| + |21 - 20.33| + |13 - 21.33| \right) / 9 = 3.184$$

MAD for b

$$MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$$

$$\left(|20 - 18.75| + |15 - 19.50| + |14 - 16.75| + |16 - 15.75| + |18 - 15.25| + |23 - 16.50| + |20 - 20| + |21 - 20.25| + |13 - 21.25| \right) / 9 = 3.000$$

d. MSE for a

$$MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$$

$$\left[(20 - 19.33)^2 + (15 - 19.33)^2 + (14 - 17.33)^2 + (16 - 16.33)^2 + (18 - 15)^2 + (23 - 16)^2 + (20 - 19)^2 + (21 - 20.33)^2 + (13 - 21.33)^2 \right] / 9 = 17.69$$

MSE for b

$$MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$$

$$\left[\frac{(20-18.75)^2 + (15-19.50)^2 + (14-16.75)^2 + (16-15.75)^2 + (18-15.25)^2 + (23-16.50)^2 + (20-20)^2 + (21-20.25)^2 + (13-21.25)^2}{9} \right] = 16.43$$

e. Use the 3-month weighted average because it was more accurate.

6. a. $F_{1991} = (2328 + 2584 + 2233) / 3 = 2381.667$

$$F_{1992} = (2584 + 2233 + 1864) / 3 = 2227.000$$

$$F_{1993} = (2233 + 1864 + 2039) / 3 = 2045.333$$

$$F_{1994} = (1864 + 2039 + 2440) / 3 = 2114.333$$

$$F_{1995} = (2039 + 2440 + 2481) / 3 = 2320.000$$

$$F_{1996} = (2440 + 2481 + 2530) / 3 = 2483.667$$

$$F_{1997} = (2481 + 2530 + 2301) / 3 = 2437.333$$

$$F_{1998} = (2530 + 2301 + 2291) / 3 = 2374.000$$

$$F_{1999} = (2301 + 2291 + 2978) / 3 = 2523.333$$

$$F_{2000} = (2291 + 2978 + 2487) / 3 = 2585.333$$

b.

$$F_{1991} = (.2)(2328) + (.3)(2584) + (.5)(2233) = 2357.300$$

$$F_{1992} = (.2)(2584) + (.3)(2233) + (.5)(1864) = 2118.700$$

$$F_{1993} = (.2)(2233) + (.3)(1864) + (.5)(2039) = 2025.300$$

$$F_{1994} = (.2)(1864) + (.3)(2039) + (.5)(2440) = 2204.500$$

$$F_{1995} = (.2)(2039) + (.3)(2440) + (.5)(2481) = 2380.300$$

$$F_{1996} = (.2)(2440) + (.3)(2481) + (.5)(2530) = 2497.300$$

$$F_{1997} = (.2)(2481) + (.3)(2530) + (.5)(2301) = 2405.700$$

$$F_{1998} = (.2)(2530) + (.3)(2301) + (.5)(2291) = 2341.800$$

$$F_{1999} = (.2)(2301) + (.3)(2291) + (.5)(2978) = 2636.500$$

$$F_{2000} = (.2)(2291) + (.3)(2978) + (.5)(2487) = 2595.100$$

c. $F_{1992} = (2328 + 2584 + 2233 + 1864) / 4 = 2252.250$

$$F_{1993} = (2584 + 2233 + 1864 + 2039) / 4 = 2180.000$$

$$F_{1994} = (2233 + 1864 + 2039 + 2440) / 4 = 2144.000$$

$$F_{1995} = (1864 + 2039 + 2440 + 2481) / 4 = 2206.000$$

$$F_{1996} = (2039 + 2440 + 2481 + 2530) / 4 = 2372.500$$

$$F_{1997} = (2440 + 2481 + 2530 + 2301) / 4 = 2438.000$$

$$F_{1998} = (2481 + 2530 + 2301 + 2291) / 4 = 2400.750$$

$$F_{1999} = (2530 + 2301 + 2291 + 2978) / 4 = 2525.000$$

$$F_{2000} = (2301 + 2291 + 2978 + 2487) / 4 = 2514.250$$

d. $F_{1992} = (.1)(2328) + (.2)(2584) + (.3)(2233) + (.4)(1864) = 2165.100$

$$F_{1993} = (.1)(2584) + (.2)(2233) + (.3)(1864) + (.4)(2039) = 2079.800$$

$$F_{1994} = (.1)(2233) + (.2)(1864) + (.3)(2039) + (.4)(2440) = 2183.800$$

$$F_{1995} = (.1)(1864) + (.2)(2039) + (.3)(2440) + (.4)(2481) = 2318.600$$

$$F_{1996} = (.1)(2039) + (.2)(2440) + (.3)(2481) + (.4)(2530) = 2448.200$$

$$F_{1997} = (.1)(2440) + (.2)(2481) + (.3)(2530) + (.4)(2301) = 2419.600$$

$$F_{1998} = (.1)(2481) + (.2)(2530) + (.3)(2301) + (.4)(2291) = 2360.800$$

$$F_{1999} = (.1)(2530) + (.2)(2301) + (.3)(2291) + (.4)(2978) = 2591.700$$

$$F_{2000} = (.1)(2301) + (.2)(2291) + (.3)(2978) + (.4)(2487) = 2576.500$$

e. $F_{t+1} = F_t + \alpha(D_t - F_t)$

$$F_{1989} = 2328.000 + .5(2328 - 2328.000) = 2328.000$$

$$F_{1990} = 2328.000 + .5(2584 - 2328.000) = 2456.000$$

$$F_{1991} = 2456.000 + .5(2233 - 2456.000) = 2344.500$$

$$F_{1992} = 2344.500 + .5(1864 - 2344.500) = 2104.250$$

$$F_{1993} = 2104.250 + .5(2039 - 2104.250) = 2071.625$$

$$F_{1994} = 2071.625 + .5(2440 - 2071.625) = 2255.813$$

$$F_{1995} = 2255.813 + .5(2481 - 2255.813) = 2368.406$$

$$F_{1996} = 2368.406 + .5(2530 - 2368.406) = 2449.203$$

$$F_{1997} = 2449.203 + .5(2301 - 2449.203) = 2375.102$$

$$F_{1998} = 2375.102 + .5(2291 - 2375.102) = 2333.051$$

$$F_{1999} = 2333.051 + .5(2978 - 2333.051) = 2655.525$$

$$F_{2000} = 2655.525 + .5(2487 - 2655.525) = 2571.263$$

f. MAD for a

$$MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$$

$$\left(|2381.667 - 1864| + |2227.000 - 2039| + |2045.333 - 2440| + |2114.333 - 2481| + |2320.000 - 2530| + \right. \\ \left. |2483.667 - 2301| + |2437.333 - 2291| + |2374.000 - 2978| + |2523.333 - 2487| \right) / 9 = 294.037$$

MAD for b

$$MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$$

$$\left(|2357.300 - 1864| + |2118.700 - 2039| + |2025.300 - 2440| + |2204.500 - 2481| + |2380.300 - 2530| + \right. \\ \left. |2497.300 - 2301| + |2405.700 - 2291| + |2341.800 - 2978| + |2636.500 - 2487| \right) / 9 = 278.956$$

MAD for c

$$MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$$

$$\left(|2252.250 - 2039| + |2180.000 - 2440| + |2144.000 - 2481| + |2206.000 - 2530| + \right. \\ \left. |2372.500 - 2301| + |2438.000 - 2291| + |2400.750 - 2978| + |2525.000 - 2487| \right) / 8 = 246.000$$

MAD for d

$$MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$$

$$\left(|2165.100 - 2039| + |2079.800 - 2440| + |2183.800 - 2481| + |2318.600 - 2530| + \right. \\ \left. |2448.200 - 2301| + |2419.600 - 2291| + |2360.800 - 2978| + |2591.700 - 2487| \right) / 8 = 249.075$$

MAD for e

$$MAD = \left(\sum_{t=1}^n |D_t - F_t| \right) / n$$

$$\left(|2328.000 - 2584| + |2456.000 - 2233| + |2344.500 - 1864| + \right. \\ \left. |2104.250 - 2039| + |2071.625 - 2440| + |2255.813 - 2481| + |2368.406 - 2530| + \right. \\ \left. |2449.203 - 2301| + |2375.102 - 2291| + |2333.051 - 2978| + |2655.525 - 2487| \right) / 11 = 256.881$$

g. MSE for a

$$MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$$

$$\left[(2381.667 - 1864)^2 + (2227.000 - 2039)^2 + (2045.333 - 2440)^2 + (2114.333 - 2481)^2 + \right. \\ \left. (2320.000 - 2530)^2 + (2483.667 - 2301)^2 + (2437.333 - 2291)^2 + (2374.000 - 2978)^2 + (2523.333 - 2487)^2 \right] / 9 = 117,616.185$$

MSE for b

$$MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$$

$$\left[(2357.300 - 1864)^2 + (2118.700 - 2039)^2 + (2025.300 - 2440)^2 + (2204.500 - 2481)^2 + (2380.300 - 2530)^2 + (2497.300 - 2301)^2 + (2405.700 - 2291)^2 + (2341.800 - 2978)^2 + (2636.500 - 2487)^2 \right] / 9 = 111,036.209$$

MSE for c

$$MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$$

$$\left[(2252.250 - 2039)^2 + (2180.000 - 2440)^2 + (2144.000 - 2481)^2 + (2206.000 - 2530)^2 + (2372.500 - 2301)^2 + (2438.000 - 2291)^2 + (2400.750 - 2978)^2 + (2525.000 - 2487)^2 \right] / 8 = 86,625.442$$

MSE for d

$$MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$$

$$\left[(2165.100 - 2039)^2 + (2079.800 - 2440)^2 + (2183.800 - 2481)^2 + (2318.600 - 2530)^2 + (2448.200 - 2301)^2 + (2419.600 - 2291)^2 + (2360.800 - 2978)^2 + (2591.700 - 2487)^2 \right] / 8 = 88,595.848$$

MSE for e

$$MSE = \left(\sum_{t=1}^n (D_t - F_t)^2 \right) / n$$

$$\left[(2328.000 - 2584)^2 + (2456.000 - 2233)^2 + (2344.500 - 1864)^2 + (2104.250 - 2039)^2 + (2071.625 - 2440)^2 + (2255.813 - 2481)^2 + (2368.406 - 2530)^2 + (2449.203 - 2301)^2 + (2375.102 - 2291)^2 + (2333.051 - 2978)^2 + (2655.525 - 2487)^2 \right] / 11 = 94,211.131$$